



# Problem C

## Canonical Coin Systems

A *coin system*  $S$  is a finite (nonempty) set of distinct positive integers corresponding to coin values, also called *denominations*, in a real or imagined monetary system. For example, the coin system in common use in Canada is  $\{1, 5, 10, 25, 100, 200\}$ , where 1 corresponds to a 1-cent coin and 200 corresponds to a 200-cent (2-dollar) coin. For any coin system  $S$ , we assume that there is an unlimited supply of coins of each denomination, and we also assume that  $S$  contains 1, since this guarantees that any positive integer can be written as a sum of (possibly repeated) values in  $S$ .

Cashiers all over the world face (and solve) the following problem: For a given coin system and a positive integer amount owed to a customer, what is the smallest number of coins required to dispense exactly that amount? For example, suppose a cashier in Canada owes a customer 83 cents. One possible solution is  $25 + 25 + 10 + 10 + 10 + 1 + 1 + 1$ , i.e., 8 coins, but this is not optimal, since the cashier could instead dispense  $25 + 25 + 25 + 5 + 1 + 1 + 1$ , i.e., 7 coins (which is optimal in this case). Fortunately, the Canadian coin system has the nice property that the *greedy algorithm* always yields an optimal solution, as do the coin systems used in most countries. The greedy algorithm involves repeatedly choosing a coin of the largest denomination that is less than or equal to the amount still owed, until the amount owed reaches zero. A coin system for which the greedy algorithm is always optimal is called *canonical*.

Picture	Name	Amount
	toonie	2 dollars
	loonie	1 dollar
	quarter	25 cents
	dime	10 cents
	nickel	5 cents
	penny	1 cent

Image by Diane Wiens, Used with permission

Your challenge is this: Given a coin system  $S = \{c_1, c_2, \dots, c_n\}$ , determine whether  $S$  is canonical or non-canonical. Note that if  $S$  is non-canonical then there exists at least one *counterexample*, i.e., a positive integer  $x$  such that the minimum number of coins required to dispense exactly  $x$  is less than the number of coins used by the greedy algorithm. An example of a non-canonical coin system is  $\{1, 3, 4\}$ , for which 6 is a counterexample, since the greedy algorithm yields  $4 + 1 + 1$  (3 coins), but an optimal solution is  $3 + 3$  (2 coins). A useful fact (due to Dexter Kozen and Shmuel Zaks) is that if  $S$  is non-canonical, then the smallest counterexample is less than the sum of the two largest denominations.

### Input

Input consists of a single case. The first line contains an integer  $n$  ( $2 \leq n \leq 100$ ), the number of denominations in the coin system. The next line contains the  $n$  denominations as space-separated integers  $c_1 c_2 \dots c_n$ , where  $c_1 = 1$  and  $c_1 < c_2 < \dots < c_n \leq 10^6$ .

### Output

Output “canonical” if the coin system is canonical, or “non-canonical” if the coin system is non-canonical.

#### Sample Input 1

```
4
1 2 4 8
```

#### Sample Output 1

```
canonical
```



### Sample Input 2

```
3
1 5 8
```

### Sample Output 2

```
non-canonical
```

### Sample Input 3

```
6
1 5 10 25 100 200
```

### Sample Output 3

```
canonical
```